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DETERMINING THE TEMPERATURE FIELDS OF MULTILAYER SPHERICALLY SYMMETRIC SYSTEMS

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On the basis of the WKBJ method, approximate relations are obtained for determining the nonsteady temperature field in spherically symmetric multilayer systems.

In calculating the temperature fields of multilayer shells, power plants, and various aircraft, not only computer-based numerical methods may be successfully used, but also approximate analytical methods with the introduction of effective thermophysical characteristics for inhomogeneous media [1-4]. It is very expedient in this case to use the principles of the WKBJ method [5], which is well developed and widely used in theoretical physics. In thermophysical investigations, these ideas have been realized in determining the temperature in plane multilayer systems [6], and estimates have been made for the cylindrical case [7].

Consider a multilayer spherically symmetric system. Its temperature field is determined by the equations

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\varkappa(r)\frac{\partial T}{\partial r}\right]+q_v(r,\ t)=c\gamma(r)\frac{\partial T}{\partial t}, \tag{1}$$

$$\frac{\partial T}{\partial r}=0,\quad r=0, \tag{2}$$

$$-\varkappa\frac{\partial T}{\partial r}=\alpha(T-T_{me}),\quad r=R, \tag{3}$$

$$T(r,\ t=0)=T_{in}(r), \tag{4}$$

where κ, $c\gamma$ are the thermal conductivity and volume specific heat; q_v denotes the heat sources; α, T_{me} are the heat-transfer coefficient with the surrounding medium and its temperature; R is the radius of the system.

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TABLE 1. Roots of Dispersion Equation for Two-Layer Sphere

Bi	K_a	n	λ_n	λ_{nT}
0	10	1	1,708	1,430
		2	2,943	2,521
		3	4,157	3,684
		4	5,364	4,905
	100	1	0,579	0,452
		2	0,996	0,799
		3	1,407	1,170
		4	1,815	1,559
0,1	10	1	1,715	1,437
		2	2,947	2,525
		3	4,159	3,687
		4	5,366	4,907
	100	1	0,0485	0,0542
		2	0,581	0,455
		3	0,997	0,800
		4	1,408	1,17

Introducing $\xi = \int_0^r [dr/\kappa(r)]$ and separating variables in the homogeneous equation (with $q_V = 0$), the following relation may be obtained for a weak dependence $\kappa(\xi)$

$$\frac{d^2\overline{T}}{d\xi^2} + \frac{2}{\xi}\frac{d\overline{T}}{\partial\xi} + c\gamma\varkappa(\xi)\lambda^2\overline{T} = 0. \tag{5}$$

The solution of this equation when $c\gamma\kappa \sim$ const is obtained by the WKBJ method

$$T(\xi) \simeq A\frac{\sin\int_0^\xi \sqrt{c\gamma\varkappa(\xi)}\,\lambda d\xi}{\xi}, \tag{6}$$

and the solution of the initial problem by the widely used method of [8, 9] gives

$$T(\xi, t) \simeq T_{me} + \sum_{n=1}^{\infty}\frac{1}{\|y_n\|^2}\left\{\int_0^{\xi_R}[T_{in}(\xi) - T_{me}]\,\xi\sin\frac{\mu_n\int_0^\xi\sqrt{c\gamma\varkappa(\xi)}\,d\xi}{\int_0^{\xi_R}\sqrt{c\gamma\varkappa(\xi)}\,d\xi}\,d\xi\right\}\times$$

$$\times\frac{1}{\xi}\sin\frac{\mu_n\int_0^\xi\sqrt{c\gamma\varkappa(\xi)}\,d\xi}{\int_0^{\xi_R}\sqrt{c\gamma\varkappa(\xi)}\,d\xi}\exp\left(-\frac{\mu_n^2 t}{\Pi^2(R)}\right) + F(q_v, \xi, t), \tag{7}$$

$$F(q_v, \xi, t) = \sum_{n=1}^{\infty}\frac{1}{\|y_n\|^2}\frac{1}{\xi}\sin\frac{\mu_n\int_0^\xi\sqrt{c\gamma\varkappa(\xi)}\,d\xi}{\int_0^{\xi_R}\sqrt{c\gamma\varkappa(\xi)}\,d\xi}\times$$

$$\times\left\{\int_0^t\exp\left(-\frac{\mu_n^2(t-\tau)}{\Pi^2(R)}\right)d\tau\int_0^{\xi_R}\frac{\xi q_v(\xi, \tau)}{c\gamma(\xi)}\sin\frac{\mu_n\int_0^\xi\sqrt{c\gamma\varkappa(\xi)}\,d\xi}{\int_0^{\xi_R}\sqrt{c\gamma\varkappa(\xi)}\,d\xi}\,d\xi\right\}, \tag{8}$$

with the addition of the following term when $\alpha = 0$

$$\frac{\int_0^R r^2 [T_{in}(r) - T_{me}]\, c\gamma(r)\, dr}{\int_0^R r^2 c\gamma(r)\, dr}, \tag{9}$$

$$\|y_n\|^2 = \frac{1}{2}\sum_{i=1}^{N}\frac{\Pi_i - \Pi_{i-1}}{\sqrt{c_i\gamma_i\varkappa_i}} - \frac{\Pi(R)}{4\mu_n}\sum_{i=1}^{N}\frac{\sin\frac{2\mu_n\Pi_i}{\Pi(R)} - \sin\frac{2\mu_n\Pi_{i-1}}{\Pi(R)}}{\sqrt{c_i\gamma_i\varkappa_i}}, \tag{10}$$

$$\xi_R = \int_0^R \frac{dr}{\varkappa(r)}; \quad \Pi_i = \int_0^{r_i}\frac{dr}{\sqrt{a(r)}}, \quad \Pi(R) = \int_0^R \frac{dr}{\sqrt{a(r)}}, \quad a = \frac{\varkappa}{c\gamma}. \tag{11}$$

Here N is the number of spherical layers; i is the layer number. The eigenvalues are determined from the characteristic equation

$$\operatorname{tg}\mu = \mu\frac{\xi_R\sqrt{c\gamma\varkappa(R)}}{(1-\xi_R\alpha)\Pi(R)}, \tag{12}$$

$$\lambda = \frac{\mu}{\Pi(R)}. \tag{13}$$

Equation (12) in the form μ ctg $\mu + C = 0$ was tabulated in [9]. The eigenvalues λ_n of the problem for a two-layer sphere obtained using Eqs. (12) and (13) and the accurate solutions of the corresponding dispersion equation λ_{nT} for various values of the parameters $Bi = (\alpha/\kappa_2)R$ and $K_a = a_1/a_2$ are shown in Table 1. Comparison of these results allows the approach proposed on the basis of the WKBJ method — the introduction of effective thermophysical characteristics — to be recommended in estimating the time to reach steady conditions and the temperature field of multilayer spherically symmetric systems.

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